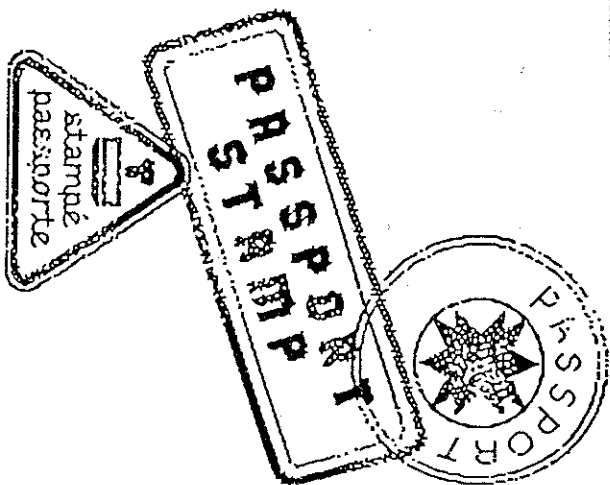


## Suggestions From Research For Improving Mathematics Instruction For Bilinguals

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Most attention by educators to bilingual instruction has been, and continues to be, directed to such language aspects of the curriculum as reading, writing, language acquisition, and communication. In the California State Department of Education publication, *Schooling and Language Minority Students: A Theoretical Framework* (1982), for example, little attention is given to mathematics instruction. Most of this text is directed to language arts and reading, even though this is a basic reference on bilingual education for a state with a large population of bilingual students.

This lack of attention to mathematics instruction for bilinguals has many causes. Most bilingual researchers and specialists are linguists and language experts who, recognizing the heavy language load placed on bilingual students and teachers, have attempted to offer help in those areas which seemed most obvious and most pressing. Further, since bilingual specialists and researchers are usually more knowledgeable about language and linguistic matters, they tend to concentrate on their own areas of specialization. A third reason has to do with the perception of many educators that mathematics is a universal language of symbols, a process that almost transcends language concerns. With pressing language problems to handle in other curricular fields, bilingual educators have often regarded mathematics as an area which deals with absolutes and universals, and which may well be one of the first areas where students can safely make the transition from their first language (L1) to their second (L2), since

2+2 always equals 4, the argument goes, in any language. Recently, however, researchers and theorists in bilingual education have turned more attention to mathematics in bilingual programs. These studies, while still scarce in relation to the numbers of studies about language acquisition and bilingual reading, are attacking such problems as the complexity of the interplay between language and mathematics skills for bilinguals and are beginning to offer guidelines for better mathematics instruction. The suggestions, while still tenuous in nature, do give starting places for designing more comprehensive and more effective bilingual mathematics programs.

### Suggestions For Improving Mathematics Instruction For Bilinguals

The following instructional procedures, which are based on a review of research on mathematics instruction for bilinguals, offer promise:

1. Develop bilingual students' first-language (L1) competence, especially to improve later mathematical ability in English.
2. Develop bilingual students' second language (L2) proficiency.
3. Teach mathematics to bilingual children bilingually.
4. Recognize the role of language in mathematical problem-solving.
5. Recognize that mathematics is not necessarily a "universal language" for bilinguals.
6. Teach mathematics vocabulary directly and systematically.
7. Consider pairing L2-dominant students with L1-dominant students for English mathematics instruction as one grouping method.
8. Teach problem-solving skills directly.
9. Understand that social-interaction patterns can affect mathematical achievement.
10. Use culturally relevant situations and illustrations.
11. Use individualized instruction and a diagnostic-prescriptive approach.
12. Provide extensive staff development, including opportunities for bilingual mathematics teachers to develop their own materials.
13. Develop a planned, parent-participation model.
14. Recognize the positive effects of bilingualism on mathematics learning.

Each of these suggestions is discussed more fully below:

1. *Develop bilingual students' first language (L1) competence, especially to improve later mathematical reasoning ability in English:* Dawe (1983) found first-language competence to be an important factor in the ability to do mathematical reasoning in English for the varied-background, bilingual British children, aged 11-13, he studied. Danker's (1977) research on primary-grade, Spanish-speaking children reinforced the importance of first-language competence as a factor in achievement in mathematics. She concluded that children who are instructed in their native language perform in accordance with their ability in the subject area, as contrasted Spanish-speaking children who receive instruction only in English, and perform in mathematics according to their English competence.
2. *Develop bilingual students' second-language (L2) proficiency:* Special and intensive attention to second-language proficiency appears to be important to mathematical achievement of bilingual students. Gerace and Mestre (1983) reported that the Hispanic undergraduate participants, in their study of barriers which impede Hispanics, make errors in solving mathematics and logics problems in English more because of semantic difficulties than because of difficulties with content processes. Mestre's (1980) study of Spanish-speaking science and engineering majors at MIT found that the mathematical performance of these bilinguals was more dependent upon language proficiency than

was that of monolinguals in the study. Denker (1977), investigating the mathematical performances of Spanish-speaking second-graders, concluded that children whose native language is not English perform according to their English competence when instructed in English. Mestre, Gerace, and Loothead (1982) found, from their study of Hispanic engineering and science majors, that their mathematical translation task was more strongly correlated with language proficiency for bilinguals than for monolinguals. Mestre's (1981) study of undergraduate engineering and science students reported that the bilingual (Spanish-English) group exhibited a persistent and statistically significant correlation between language-proficiency measures and scores on mathematics examinations which required virtually no language processing; a similar correlation was not found for monolinguals. Gerace and Mestre (1981), who studied bilingual Hispanics in technical fields, reported that these students' technical skills (e.g., the ability to translate verbal statements into mathematical equations, and the reverse) were more strongly correlated with linguistic skills than were those of monolinguals.

3. *Teach mathematics to bilingual children bilingually:* The importance to mathematical achievement for instruction to take place bilingually has been underscored by several research studies. Bacon, Kidd, and Seaberg (1982) reported that the mathematical achievement of Cherokee eighth-graders improved for those who had received four to five years bilingual instruction over those Cherokee students who had been instructed entirely in English. Rosier and Holm (1980) found that the bilingual Navajo elementary-school children in their study who had received bilingual instruction scored higher on standardized achievement tests, including mathematics, than did students who had received English-language-only instruction. Additionally, they found that bilingual students who were taught mathematics in Navajo and English until the end of second grade scored significantly higher in mean scores on the Total Arithmetic subsists than did Navajo students taught in English.

4. *Recognize the role of language in mathematical problem-solving:* The interplay between language and mathematics tasks is of special importance to bilinguals, according to research. Coffland and Cuevas (1979) studied the learning of mathematical concepts by English-dominant and Spanish-dominant, bilingual, primary-age children. Among other results, they found that children's language skills had not completely developed by first grade and that there were specific usages that occur in mathematics teaching which cause children difficulty. Gerace and Mestre (1983) found similar kinds of results in their studies of critical barriers to achievement for undergraduate bilingual science and engineering students. The underlying theme they reported was the intricate interplay among language and various problem-solving tasks, with semantic difficulties more important to problem-solving achievement than was knowledge of the content of mathematics. In sum, errors caused by misinterpretation of the problem statement were more crucial than errors caused by mathematical deficiencies for these students.

5. *Recognize that mathematics is not necessarily a "universal language" for bilinguals:* While mathematical symbols are fairly universal across cultures, there are differences. Proof processes, division set-ups, and commas and periods in number notation, for instance, are different in Spanish and English. The mental processes required for division, as taught in Latin American countries, for example, and those processes taught in the United States, while similar in results, require quite different notations, problem-solving steps, and mental tasks to be done without using pencil and paper. The depth of mathematics vocabulary and concepts necessary for students who have learned division in this manner to explain such differences to U.S.-trained teachers is, consequently, of even greater complexity than knowledgeable, monolingual students

would typically have. Differences occur, as well, in these mathematical processes in other languages.

Dew (1981), in her summary of materials, processes, and programs for learning-disabled, elementary-school bilinguals stressed the myth of mathematics as a universal language. Mestre (1981) found that, for the bilingual undergraduate science and engineering students in his study, even on mathematics examinations which required almost no language processing, there was a persistent and statistically significant correlation with language-proficiency measures; no such correlation was found for the monolingual students studied.

6. *Teach mathematics vocabulary directly and systematically:* The importance of a deliberate and systematic attention to both native and second-language mathematics vocabulary is supported by research. Gallegos (1978) developed and evaluated a model for training bilingual (Spanish-English) mathematics teachers. He concluded from his research that instruction which emphasized basic Spanish mathematical vocabulary for each topic was more effective than that which did not. Coffland and Cuevas (1979), who studied English-dominant and Spanish-dominant, primary-age children, found that there were specific language usages which occur in mathematics and cause children difficulty, especially since primary children's language skills are not completely developed.

One conclusion from Gallegos' (1978) evaluation study of the effectiveness of a mathematics training program for bilingual teachers was that mathematics instruction in the elementary school involves much more than the four operations of addition, subtraction, multiplication, and division. This conclusion is supported, as well, by such publications on mathematical curriculum for monolinguals as the *Mathematics Framework*, published by the California State Department of Education (1985). This widely-used framework is typical in that it stresses problem solving/applications as the overall umbrella for seven strands of mathematics (arithmetic numbers and operations, geometry, measurement, calculators/computers, probability and statistics, relations and functions, and logical thinking) which run through each year of the mathematics curriculum, grades kindergarten through twelve. Such a mathematics program imposes a higher linguistic load on children, as it emphasizes higher-level thinking and process skills which are dependent on increasingly sophisticated levels of mathematical vocabulary.

7. *Consider pairing L2-dominant students with L1-dominant students for English mathematics instruction, as one grouping method:* Among other grouping methods, the pairing of L2-dominant students with L1-dominant students appears to be a promising teaching procedure. Gordon (1976) studied such groupings for Puerto Rican junior-high school students in bilingual programs. He found that English-dominant students paired with Spanish-dominant students achieved significantly better mathematics scores than did English-dominant students paired with English-dominant students. The same was true for Spanish-dominant students paired with English-dominant students, when compared with Spanish-dominant pairings.

8. *Teach problem-solving skills directly:* Research studies in bilingual settings are beginning to record evidence similar to that already reported for monolingual mathematics instruction: problem-solving skills in bilingual classrooms need to be taught directly and systematically. Mestre, Gerace, and Loothead (1982), for example, after studying problem-solving skills of undergraduate Hispanic engineering and science majors, suggested that a problem-solving course could lead to marked improvement in the mathematics performance of bilingual students.

9. *Understand that social-interaction patterns can affect mathematical achievement.* That social-interaction patterns can affect mathematical achievement is evident from such research studies as that of Coffland and Cuevas (1979). Of the English-dominant and Spanish-dominant, primary-age, bilingual students they studied, those who were more field-independent (less influenced by persons and objects in the learning milieu) tended to achieve higher in mathematics, while those who were more field-sensitive (more sensitive to persons and objects) tended to be associated with lower levels of achievement. Further, field-independent students tended to perform better on precise thinking exercises and demonstrated higher ability to use and apply mathematical rules. For many minority children, who tend to be more field-dependent than majority-culture students, high mathematical achievement may be seen as a betrayal of cultural social-interaction patterns because their cultural patterns reinforce group values and behaviors rather than individual achievement. Bilinguals may need help to deal with this interplay between mathematical achievement, and such differing cultural patterns of interaction; and they may need help to achieve scholastically without suffering group punishment. As well, they may need teacher assistance in learning field-independent behaviors which may cause them to better their mathematical achievement. Ramirez and Castaneda (1974) reviewed the field-independent, field-dependent studies on minority children and made detailed, comprehensive suggestions of culturally specific ways to improve academic achievement.

10. *Use culturally relevant situations and illustrations.* The bilingual mathematics instructional materials developed, tested, and evaluated by the Dissemination and Assessment Center for Bilingual Education, Austin, Texas, (Example: 1978) stressed the use of culturally relevant materials which reinforce bilingual students' self-concept and encourage cultural pride. Learning about the mathematical contributions of the Maya and how mathematics fits into the Hispanic community are examples of the types of materials developed from this Center's synthesis and review of the then-current research on bilingual mathematics as a basis for curriculum development. Ramirez and Castaneda (1974) offer other specific suggestions for using culturally relevant situations to promote academic achievement, in their exhaustive review of research on bicultural development of bilinguals.

11. *Use individual instruction and a diagnostic-prescriptive approach.* Kahn's work (1976) is an example of studies which found individualized mathematics instruction helpful for bilinguals. He suggested that the use of individualized instruction as the chief teaching method and the use of a diagnostic-prescriptive approach were among the factors which contributed to highly significant gains in reading and mathematics for the bilingual ninth-to-twelfth graders in his study.

12. *Provide extensive staff development, including opportunities for bilingual mathematics teachers to develop their own materials.* Gallegos (1978) and Tobias (1982) found that teachers needed extensive staff development to manage successful bilingual mathematics programs. Gallegos developed and evaluated intensive teacher-training programs which focused on helping teachers learn that successful teaching of bilingual mathematics involved more than four basic rote operations and monotonous drudgery. His successful programs included technical vocabulary development, Spanish-language lecture-demonstrations, practice presentations, feedback regarding results, rationale underlying each program component, and mathematics theory and concept development. Tobias concluded that staff development and curriculum and materials development were important components for the successful skill-development program for the mildly-retarded Spanish-speaking elementary and secondary students in his

research. In both instances, the chance to develop mathematics curriculum and materials promoted staff understanding of special components of bilingual mathematics instruction.

13. *Develop a planned, parent-participation model.* Tobias (1982) concluded that parent-involvement activities were important to project success in his study of his successful skills-development program for the bilingual student subjects described above. Spriddakis (1982) reported similar, successful use of parent participation in a study of a program for handicapped, Hispanic elementary and intermediate-school students. Program-parent communication was maintained through parent workshops and home visits.

14. *Recognize the positive effects of bilingualism on mathematics learning.* Fradd (1982) as a consequence of her reviews of research on bilingual achievement patterns, concluded that bilingualism had positive effects on cognitive growth and divergent thinking (and, incidentally, she encouraged U.S. educators to consider bilingual education as a curriculum model for non-bilinguals). In addition, she cited evidence which indicated that bilinguals have the potential for increased communication and greater problem-solving skill than do non-bilinguals. Coffland and Cuevas (1982) reported that bilingual, Hispanic students tended to perform at higher skills levels than did either English or Spanish-monolingual students, when grouped into English-dominant and Spanish-dominant pairings, as compared to English-English and Spanish-Spanish dominant pairings. Both Fradd's and Coffland and Cuevas' studies concluded that the effects of bilingualism were positive.

#### CONCLUSIONS

Ortiz-Franco (1980) pointed out that, in 1980, there was a general lack of replication of research studies with bilingual and minority-culture students of findings and practices from mathematical research on majority-culture students. He wrote that the absence of such replications added to the dubiousness of purportedly generalized outcomes. His warning is still valid today, in spite of increased attention to research on bilingual mathematics. Similarly, research has not yet clarified the interactions of two languages on children's mathematics learning and the impact of various teaching strategies on successful mathematics skills development in bilinguals. In addition, much of the research on bilingual mathematics is conducted on diverse populations. It is difficult (and often inappropriate) to generalize from bilingual Navajo to Puerto Rican to European to Chicano populations. Mathematics practitioners would do well to remember that the research is, at this point, suggestive, not conclusive, regarding the best instructional procedures.

However, bilingual educators do not have the luxury of waiting for definitive research results. Faced as they are with students who need mathematical skills and with incomplete research evidence, they must act as thoughtfully as they can on the evidence currently available, and add to and discard from their teaching methodologies as research evolves.

There is now enough research to permit an informed beginning.

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# # #

### Self Hypnosis Helps Cancer Therapy

Self-hypnosis can help young cancer patients escape some of the harrowing side effects of chemotherapy, researchers say, but the technique does not work for everyone.

"Results vary," said Pat H. Cotanch, Ph.D., "which is not surprising since each patient tends to respond differently to cancer and chemotherapy."

Ms. Cotanch is an associate professor of nursing at Duke University Medical Center and a member of the Duke Comprehensive Cancer Center.

She and co-investigator Marilyn Hockenberry have documented some dramatic successes in the initial phase of a three-year study on the potential of relaxation training to help children and young adults cope with cancer therapy.

In an interview, Cotanch said the technique has helped on average three out of five patients, decreasing the nausea and vomiting often associated with chemotherapy. The young patients in the study have ranged in age from 9 to 21.

# # #

### Man's Oldest Known Ancestor?

The female skull of a 32-million-year-old ape regarded as man's oldest known ancestor has been reconstructed by specialists at the Duke University Primate Center. Casts made from the female skull and that of a male discovered last fall clearly reveal major details of the brain structure of Aegyptopithecus, a primate ten times older than the first humans, said Dr. Elwyn L. Simons, primate center director.

The casts indicate the cat-sized ape already had a brain rivaling the complexity of that of some modern South American monkeys, Simons said.

Both Aegyptopithecus skulls, about 4 inches long and 2 inches wide, are those of mature individuals, he said. They were found about 250 yards apart in Oligocene epoch rocks dated at about 32 million years by the potassium-argon method.

The female skull came to light when researchers noticed shattered teeth in a block of stone. The skull was found during Simons' 1985 expedition in the barren Fayum Depression, about 60 miles southwest of Cairo, Egypt. Because it was badly crushed during fossilization, the skull took more than a year to reconstruct.

The male skull, which is being painstakingly removed from its covering rock, was found last fall in an expedition sponsored by the National Science Foundation and the Geological Survey of Egypt.